STABILITY ENVELOPE - NEW TOOL FOR GENERALISED STABILITY ANALYSIS

Tapani Närhi[1] **and Martti Valtonen**[2]

[1] European Space Agency, ESTEC, PO Box 299, 2200 AG Noordwijk, The Netherlands
[2] Helsinki University of Technology, Circuit Theory Laboratory, Espoo, Finland

Abstract

A new Nyquist-type method for stability analysis is presented. The envelope of the mapping from source and load impedance planes with "normalised determinant function" is determined for all passive terminations. The circuit is unconditionally stable if the origin is not encircled or included within the envelope. A single plot reveals instabilities, caused either by internal poles of the circuit or by arbitrary terminating impedances.

1. Introduction

Conventional stability analysis methods using e.g. *k*- or μ-factors are not sufficient to reveal "hidden" instabilities that arise from internal feedback loops, e.g., in an MMIC power amplifier that contains several parallel-coupled stages. Investigation of the location of the zeros of the system determinant, i.e. Nyquist-type of stability analysis is required in such cases [1,2,3]. A linear system is stable if and only if all the zeros of its determinant lie in the left half plane (LHP), provided that none of the individual elements of the network has poles in the RHP. Platzker & al. [1] have presented such a method to determine the number of zeros in the RHP using the normalised determinant function:

$$F = \frac{\Delta}{\Delta_0} \qquad (1)$$

where Δ is the determinant of the circuit under investigation and Δ_0 is the determinant of the companion circuit which is identical to the circuit of interest except that all the dependent active sources have been set to zero. The number of zeros in the RHP is obtained by plotting function F at all frequencies and counting the number of times the locus encircles the origin.

The method of Platzker is effective in finding the "hidden" instabilities, caused by the zeros of the determinant in the RHP. However, the instabilities caused by the termination impedances still have to be analysed separately, using the conventional stability factors. If the circuit is large, containing several cascaded stages, this analysis is inconvenient as each stage has to be analysed separately (Fig.1). In addition, especially in MMICs, several stages may have common components (e.g. in bias lines), so that the borderline between stages is not well defined [4]. It would be very useful for a practical design work to have available a single stability analysis tool that is capable of detecting both types of instabilities automatically and in a single pass. The purpose of this paper is to present such a tool.

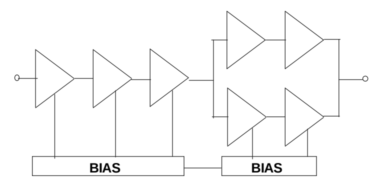

Fig. 1 Multi-stage amplifier - difficult case for stability analysis.

2. Theory

We start from a general *n*-port terminated at the input and output ports with arbitrary passive admittances Y_S and Y_L (Fig. 2).

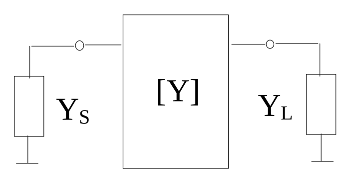

Fig. 2 General *n*-port

The *n*-port has nodal admittance matrix Y:

$$Y = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1N} \\ y_{21} & y_{22} & \cdots & y_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ y_{N1} & \cdots & \cdots & y_{NN} \end{bmatrix} \qquad (2)$$

Assigning nodes j and k to the input and output ports, we obtain admittance matrix Y' for the complete circuit:

$$Y'=\begin{bmatrix} y_{11} & \cdots & y_{1j} & \cdots & y_{1k} & \cdots & y_{1N} \\ \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ y_{j1} & \cdots & y_{jj}+Y_S & \cdots & y_{jk} & \cdots & y_{jN} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ y_{k1} & \cdots & y_{kj} & \cdots & y_{kk}+Y_L & \cdots & y_{kN} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ y_{N1} & \cdots & y_{Nj} & \cdots & y_{Nk} & \cdots & y_{NN} \end{bmatrix} \quad (3)$$

In order the circuit to be unconditionally stable, determinant Δ' of this matrix should have no zeros in the RHP with any combination of passive terminations Y_S and Y_L. We can express the determinant in the following form:

$$\Delta' = \Delta + Y_S\Delta_{jj} + Y_L\Delta_{kk} + Y_S Y_L \Delta_{jj,kk} \quad (4)$$

where $\Delta = \det(Y)$, and Δ_{jk} is a cofactor of Δ, obtained by deleting row *j* and column *k* from the matrix.

Next step is to evaluate, in the similar fashion, determinant Δ_0 (and the corresponding cofactors) of the companion circuit, where all the controlled sources have been set to zero. The normalised determinant function *F* of Eq. (1) can then be written as:

$$F = \frac{a_1\Gamma_S + b_1\Gamma_L + c_1\Gamma_S\Gamma_L + d_1}{a_2\Gamma_S + b_2\Gamma_L + c_2\Gamma_S\Gamma_L + d_2} \quad (5)$$

where:

$$\begin{cases} a_1 = -\Delta_{jj} + \Delta_{kk} - \Delta_{jj,kk} + \Delta \\ b_1 = \Delta_{jj} - \Delta_{kk} - \Delta_{jj,kk} + \Delta \\ c_1 = -\Delta_{jj} - \Delta_{kk} + \Delta_{jj,kk} + \Delta \\ d_1 = \Delta_{jj} + \Delta_{kk} + \Delta_{jj,kk} + \Delta \end{cases} \quad (6a)$$

and:

$$\begin{cases} a_2 = -\Delta^0_{jj} + \Delta^0_{kk} - \Delta^0_{jj,kk} + \Delta_0 \\ b_2 = \Delta^0_{jj} - \Delta^0_{kk} - \Delta^0_{jj,kk} + \Delta_0 \\ c_2 = -\Delta^0_{jj} - \Delta^0_{kk} + \Delta^0_{jj,kk} + \Delta_0 \\ d_2 = \Delta^0_{jj} + \Delta^0_{kk} + \Delta^0_{jj,kk} + \Delta_0 \end{cases} \quad (6b)$$

and the normalised terminations are expressed with the corresponding reflection coefficients:

$$Y_S = \frac{1-\Gamma_S}{1+\Gamma_S} \quad \text{and} \quad Y_L = \frac{1-\Gamma_L}{1+\Gamma_L} \quad (7)$$

Stability of the circuit can be investigated by plotting Eq. (5) for all Γ_S and Γ_L within a unit circle, i.e. by writing:

$$\Gamma_S = e^{j\alpha} \quad \text{and} \quad \Gamma_L = e^{j\beta} \quad (8)$$

and plotting function *F* at all frequencies for all angles α and β from 0 to 2π. If the plot does not encircle or include the origin, we can conclude that the circuit is unconditionally stable, i.e. its determinant has no zeros in the RHP for all passive terminations.

It is very cumbersome to calculate the value of determinant function *F* numerically for several frequencies and angles α and β. Therefore, we determine the envelope of *F* by calculating, for each value of angle α_0, the corresponding value of angle β that gives a point on the envelope of the plot. In this way the burden in plotting is significantly reduced, as only a single angle variable has to be swept. The envelope can be found by considering mapping $F(\alpha,\beta)$ and noting that, at each point of the envelope, $\partial F/\partial\alpha$ and $\partial F/\partial\beta$ have to be parallel [5]:

$$\arg\left(\frac{\partial F}{\partial \alpha}\right) = \arg\left(\frac{\partial F}{\partial \beta}\right) \quad (9)$$

The solution of Eq. (9) is straightforward, but tedious. The resulting equations are too long to be included here, but are given in [6]. Main requirement for implementing this generalised stability analysis in a circuit simulation programme is that the circuit determinant has to be available. The necessary modifications have been implemented in the circuit simulation software APLAC [7,8]. An example of the use of the method is shown in the next paragraph.

3. Example: Platzker's ring oscillator

The simple ring oscillator circuit of Fig. 3 was studied in detail in [1,2], and it is also used here to illustrate the stability analysis principle of this paper. With the resistor value R_2 = 5 Ω, the circuit can be either stable or unstable, depending on the terminating admittances. This was shown in [2], and also in Fig. 4, by plotting determinant function *F* for a few ideal terminations over all frequencies.

In order to determine the stability of the circuit under any terminating impedances, determinant function *F* is calculated from Eqs. (5) - (8) by numerically sweeping angles α and β from 0 to 2π at a single frequency (1.25

GHz). The result is shown in Fig. 5. The same information is obtained from the corresponding envelope of F. Fig. 6 shows the stability envelope as calculated from Eq. (9).

For complete determination of the stability of the circuit, the frequency has to be swept over a sufficient range. Fig. 7 shows the stability envelope for frequencies 0.25 GHz to 5 GHz. The circuit is unstable with most of the terminating loads, and we can see that the cases calculated in [1,2] and in Fig. 4 are special cases, and well within the envelope. Finally, Fig. 8 shows the envelope of a marginally stable case, obtained with the value g_{m2} = 0.1 S in the ring oscillator of Fig. 3.

4. Conclusion

A method for generalised stability analysis is presented. Using stability envelope, the method is capable of detecting automatically and in a single pass, instabilities of the complete circuit, whether they are caused by internal poles of the circuit or external terminations. It is believed that this method will be a valuable tool for a designer, as in the past the stability had to be analysed in several steps, including partitioning of a complex multi-stage circuit into separate parts.

References

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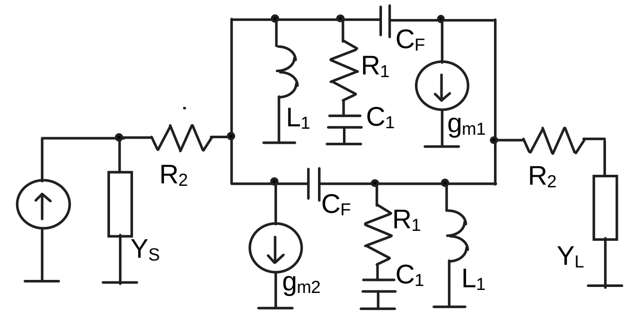

R_1 = 10 Ω	**L_1** = 560 pH	**C_1** = 16 pF
C_F = 0.1 pF	**g_{m1}** = 500 mS	**g_{m2}** = 400 mS

Fig. 3 Ring oscillator, R_2 = 5Ω [1,2].

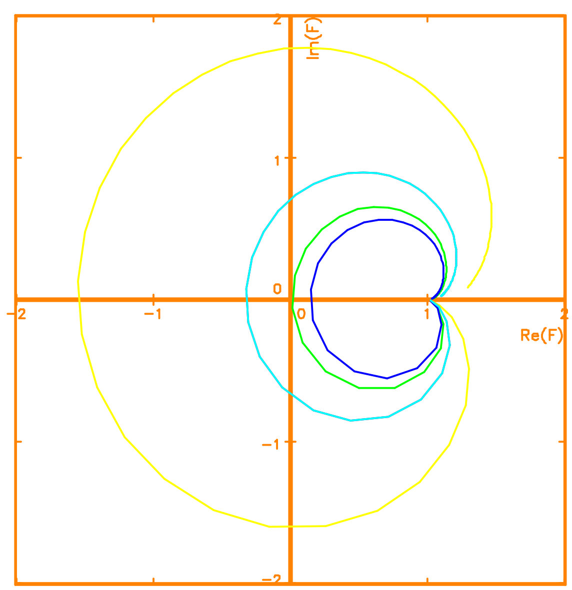

Fig. 4 Determinant function F of the ring oscillator with ideal terminations (50 Ω, short, open).

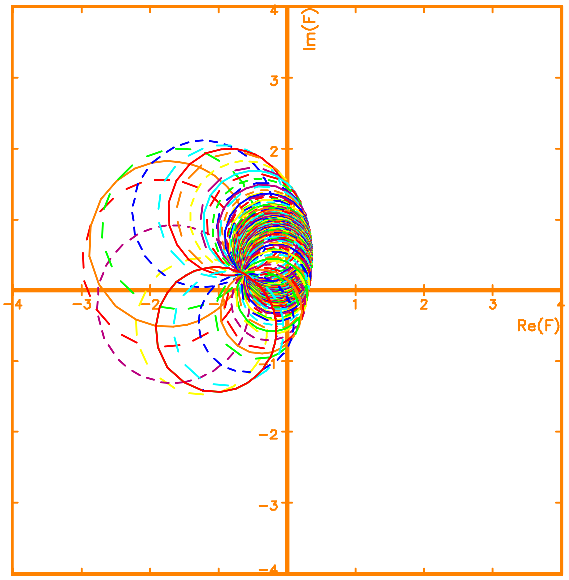

Fig. 5 *F* at a single frequency (1.25 GHz), obtained numerically from Eq. (5).

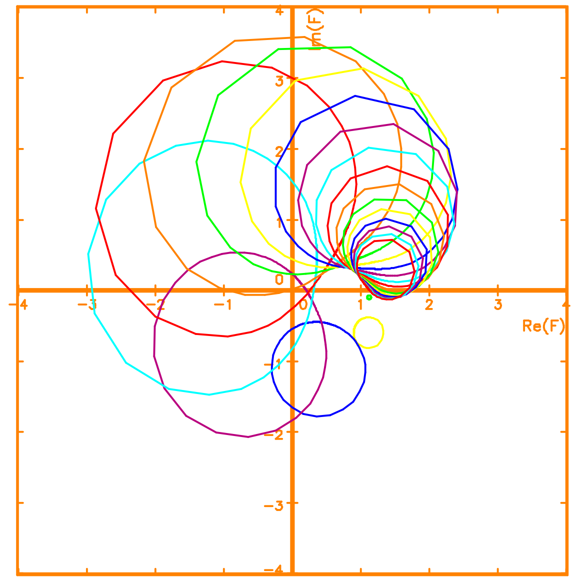

Fig. 7 Stability envelope from 0.25 to 5 GHz.

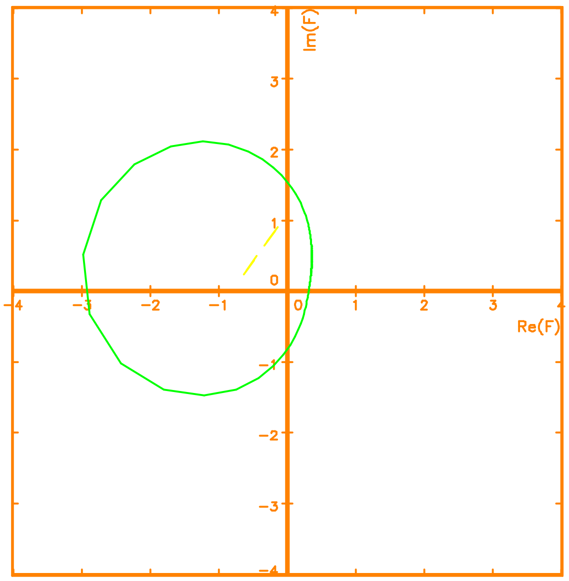

Fig. 6 Stability envelope at 1.25 GHz, calculated with the solution of Eq. (9).

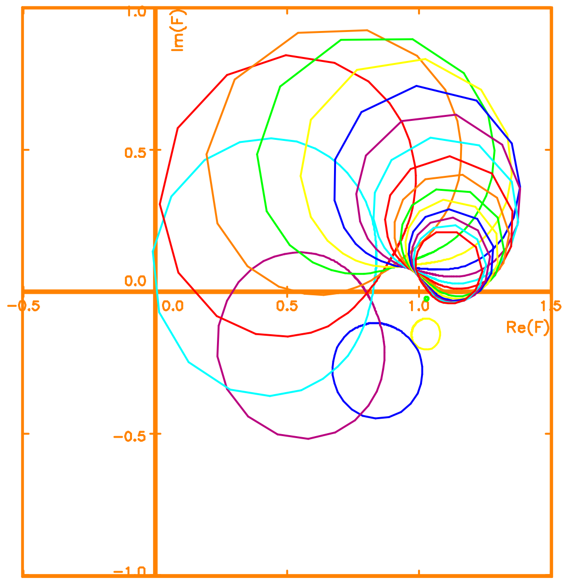

Fig. 8 Stability envelope with g_{m2} = 0.1S (from 0.25 to 5 GHz).